Lecture 2

Tuesday, November 03, 2009
10:33 AM

A. Trig. Review

Euler: $e^{jx} = \cos x + j \sin x$

$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$
$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

Can use these to derive many formula.

Example:

$$\cos A \cdot \cos B = \frac{1}{2}\left(e^{jA} + e^{-jA}\right) \times \frac{1}{2}\left(e^{jB} + e^{-jB}\right)$$
$$= \frac{1}{4}\left(\underbrace{e^{j(A+B)} + e^{-j(A+B)}}_{2\cos(A+B)} + \underbrace{e^{j(A-B)} + e^{-j(A-B)}}_{2\cos(A-B)}\right)$$
$$= \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$A = B$

$$\cos^2 A = \frac{1}{2}\left(\cos(2A) + 1\right)$$
$$\sin^2 A = 1 - \cos^2 A = 1 - \frac{1}{2}\cos 2A - \frac{1}{2} = \frac{1}{2} - \frac{1}{2}\cos 2A$$

B. Fourier Transform: Given $x(t)$

$$X_1(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$
$$X_2(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$\rightarrow \omega = 2\pi f$

Q: What is the relationship btw. $X_1(f)$ and $X_2(\omega)$?

$$X_1(f) = X_2(\omega)\Big|_{\omega = 2\pi f} = X_2(2\pi f)$$
$$X_2(\omega) = X_1(f)\Big|_{f = \frac{\omega}{2\pi}} = X_1\left(\frac{\omega}{2\pi}\right)$$

<u>Example</u> Suppose we know that

$$e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega-\omega_0)$$

$$\Downarrow$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\underline{\underline{2\pi}} f - \underline{\underline{2\pi}} f_0)$$

$$= \delta(f-f_0)$$

Remark : $X(f)$ is usually complex-valued.

Suppose $x(t)$ is real-valued.

$$X(f) = \int x(t)\, e^{-j2\pi f t}\, dt$$

$$X(-f) = \int x(t)\, e^{-j2\pi(-f)t}\, dt$$

$$= \int x(t)\, e^{(-)(-)j2\pi f t}\, dt$$

Conjugate real valued.

$a+jb$

$= (a+jb)^*$

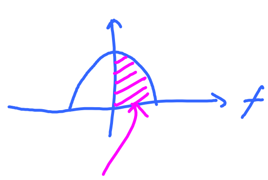
$= a-jb$

$$= \int x^*(t) \left(e^{-j2\pi f t}\right)^* dt$$

$$= \left(\int x(t)\, e^{-j2\pi f t}\, dt\right)^*$$

$$= \left(X(f)\right)^*$$

$x(t)$ real valued $\xrightarrow{\mathcal{F}} X(f)$



Positive-frequency part of the spectrum contains all the necessary information.

More Fourier transform properties

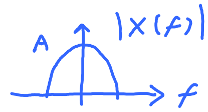
Suppose $g(t) \xrightarrow{\mathcal{F}} G(f)$.

Time-shift : $g(t-t_1) \xrightarrow{\mathcal{F}} e^{-j2\pi f t_1} G(f)$

Frequency-shift : $e^{j2\pi f_1 t} g(t) \xrightarrow{\mathcal{F}} G(f-f_1)$

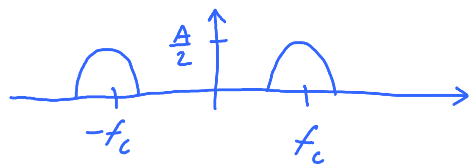
Frequency-shift: $e^{j2\pi f_1 t} g(t) \xrightarrow{\mathcal{F}} G(f - f_1)$

Example Suppose $x(t) \xrightarrow{\mathcal{F}} X(f)$



Then,

$$x(t)\cos(\omega_c t + \phi) \xrightarrow{\mathcal{F}} \frac{1}{2} e^{j\phi} X(f - f_c) + \frac{1}{2} e^{-j\phi} X(f + f_c)$$

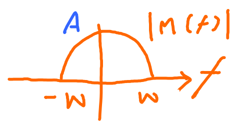


C: Modulation

Consider signal $m(t)$ that you want to transmit.

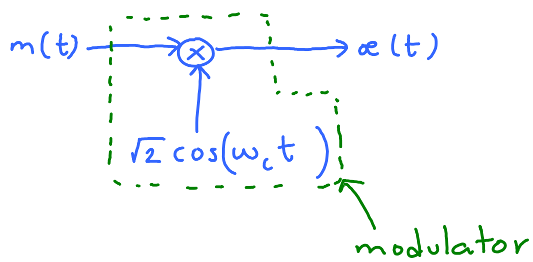
$m(t)$ is called a baseband signal.

Suppose $m(t) \xrightarrow{\mathcal{F}} M(f)$ is band-limited.

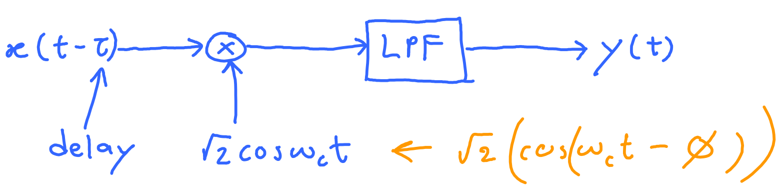


$M(f) = 0$ when $|f| > W$

We transmit $x(t) = m(t)\sqrt{2}\cos(\omega_c t)$ ←



At receiver



$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(\omega_c(t-\tau))$$

$$= m(t-\tau)\sqrt{2}\cos(\omega_c t - \underbrace{\omega_c \tau}_{\phi})$$

$$= m(t-\tau)\sqrt{2}\cos(\omega_c t - \phi)$$

$$= m(t-\tau)\sqrt{2}\cos(\omega_c t - \phi)$$

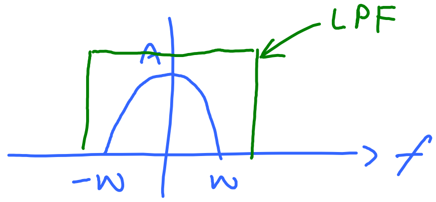
$$x(t-\tau)\sqrt{2}\cos(\omega_c t)$$

$$= m(t-\tau)\, 2\cos(\omega_c t)\cos(\omega_c t - \phi)$$

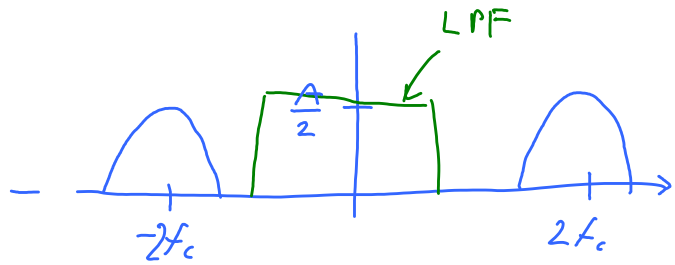
$$= m(t-\tau)\big(\cos(2\omega_c t - \phi) + \cos(\phi)\big)$$

$$= m(t-\tau)\cos(2\omega_c t - \phi) + m(t-\tau)\cos\phi$$

$$m(t-\tau) \xrightarrow{\mathcal{F}}$$



$$m(t-\tau)\cos(2\omega_c t - \phi) \xrightarrow{\mathcal{F}}$$



$$y(t) = m(t-\tau)\cos\phi = \big(m(t-\tau)\big)\cos(\omega_c \tau)$$

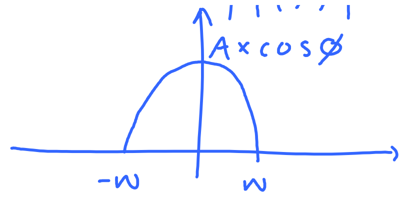
$$\cos(\theta) = 0$$

$$2\cancel{\pi} f_c \tau = \omega_c \tau = \theta = \frac{\cancel{\pi}}{2}, \frac{3\cancel{\pi}}{2}, \frac{5\cancel{\pi}}{2}, \ldots$$

$$\tau = \frac{1}{4f_c}, \frac{3}{4f_c}, \frac{5}{4f_c}$$

$$\tau = \frac{d}{c}$$

$|y(t)|$


A × cos ∅
-w
w